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THE EFFECTS OF WIND AND TEMPERATURE GRADIENTS ON SONIC BOOM CORRIDORS

by

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SUMMARY

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The effects of aircraft flight altitude and Mach number, wind and temperature gradients, and wind direction on sonic boom corridor are investigated in detail, including the effects of non-standard atmospheres such as prevail in winter months.

Numerical results are presented and amply discussed. Agreement of the present calculations with published data is excellent.

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LIST OF SYMBOLS

c	Speed of sound
D	Wind direction with respect to true North
H	Aircraft heading with respect to true North
k	Snell's constant
ℓ	ξ-direction cosine
M	Mach number
$\bar{N}$	Wave normal or phase velocity vector
n	Z-direction cosine
$\bar{n}$	Unit vector along wave normal direction
r	Cylindrical coordinate
s	Distance measured along the ray
$\bar{\dot{s}}$	Ray or group velocity vector
$\bar{\dot{s}}_\ell$	Horizontal projection of $\bar{\dot{s}}$
$\bar{\dot{s}}_v$	Project of $\bar{\dot{s}}$ on wave normal plane
t	Time
u	Wind component along ξ
u'	As defined in Eq. 3
V	Wind component along η
$\bar{W}$	Wind vector
$\bar{W}_v$	Projection of $\bar{W}$ on wave normal plane
x,y,z	Aircraft reference coordinate
$\dot{z}_r$	As defined in Eq. 5

GREEK SYMBOLS

α	Wind gradient
β	Sound gradient
ξ, η	Coordinates aligned with wave normal direction
ϕ	Wave normal azimuthal angle

γ	Angle defining wave normal plane
θ	Wave normal inclination to the horizontal
μ	Mach angle
μ'	Complement of the Mach angle
ω	H-D

SUBSCRIPTS

o	no wind condition
h	conditions at aircraft flight altitude
g	conditions at the ground level
x	component along x-axis
y	component along y-axis

1.0 INTRODUCTION

The thought of possible flights of supersonic transports (SST's) has made it increasingly necessary to ascertain accurately and reliably the lateral extent of sonic boom on either side of the flight track - not necessarily the same - in the presence of wind and temperature gradients. This type of information is necessary in planning the location of possible SST routes so that heavily populated areas are outside the sonic boom corridor. The width of the sonic boom corridor is limited by the normal atmospheric temperature gradients which bend or refract the sound rays; outside a certain width these rays do not reach the ground, thus producing a sound 'shadow' from there on out. Wind patterns modify the refraction of the rays. Calculation of the corridor width (and lateral shift in case of a side wind) depend heavily on the theory of geometrical acoustics.

Classical theories of geometrical acoustics have been developed in detail in Refs. 1, and 2. A detailed analysis, and to some extent synthesis of established theories for sonic boom propagation in a horizontally stratified atmosphere with winds was presented in Ref. 3. The analysis, though similar in some respects to the present calculations, is far too complicated for easy application and the emphasis in that report is on pressure signatures rather than corridors.

Kane, et al, Ref. 5, applied the theory of Ref. 4 to predict the variations in overpressure which would occur on the ground as a result of the variations in the atmospheric properties between the airplane and the ground. Although considerable information is available in this report, it is difficult to extract it because in many cases, the parameters for which the curves were plotted were missing.

Calculations similar to those of Ref. 5 but without winds were made earlier by Randall, Ref. 6. Dressler, et al, Ref. 7 calculated ray-ground intersections in the presence of wind and temperature gradients, using Snell's law. The method involves slicing the medium into piecewise-constant thin layers for temperature, wind magnitude and direction, and applying Snell's law for the refractive behaviour at the discontinuities at each interface. Thus, the method employs straight line rays and constant wind magnitude and direction within each layer. While the technique is basically correct, it is less realistic and less accurate than a solution employing linear variation of vector wind and temperatures between given points. Reed, et al, Ref. 8, used an acoustic ray tracing procedure to determine the ground-level sound patterns for straight, level supersonic flight in arbitrary weather conditions. The authors however treated the ray as the orthogonal trajectory of the wave front - a procedure which would be correct in the absence of winds. For small wind velocities, the error involved in this procedure is small, but for moderate and strong winds the procedure leads to totally unacceptable results.

It is thus desirable that a simple, accurate, set of calculations of sonic boom corridors in the presence of wind and temperature gradients for aircraft in steady level flight be carried out with the aim of providing practical information needed for planning the possible location of SST routes. Such calculation is the prime objective of this study. In the past, most investigators have used the components of wind velocity along the aircraft axes as the variable wind parameters. Since however, the instrument panel of the aircraft will presumably provide such data as aircraft heading, Mach number, altitude, wind

magnitude and direction, it serves a useful practical purpose to use wind magnitude and direction as the variable parameters of the problem. This approach is followed in the present study. The effects of altitude, Mach number, temperature distribution, wind magnitude and direction, on sonic boom corridor are investigated. In the mathematical analysis, a piecewise linear atmospheric model is assumed in order to permit a closed form solution. The wind direction is permitted to vary from altitude to altitude. The analysis will neither include the effects of atmospheric turbulence nor the effects of aircraft maneuvers. The latter effects will be taken up under a separate study.

2.0 MATHEMATICAL THEORY AND ANALYSIS

The equations used for calculating the propagation of sonic disturbance from a supersonic aircraft are based on the laws of geometrical acoustics. These laws state, in part, that a wave front carrying a disturbance from a surface (in this case, the aircraft body) of arbitrary shape moves such that its normal velocity relative to the medium is the undisturbed speed of sound. The normals are the orthogonal trajectories of the successive positions of the wave front, and the 'rays' may be thought of as carrying the disturbance. In a quiescent medium, the ray and the wave normal are coincident, but in a moving medium (such as atmosphere with winds)the ray and the wave normal are different. However, the horizontal component of the wave normal velocity vector remains invariant along the ray.

In general, the envelope of the acoustic disturbances herein after referred to as the wave front (attached to the aircraft at the reference point on the aircraft-the nose) does not have the form of a regular Mach cone except in the special case of straight flight at constant speed in an atmosphere of uniform temperature. The wave front and the rays are shown schematically in Fig. 1(a) for homogeneous and inhomogeneous atmospheres. The calculation of the geometry of the rays as the disturbances propagate along them from the aircraft to the observer at the ground is often called 'Ray Tracing', and the appropriate ray tracing equations will be presented in the next section.

2.1 Acoustic Ray-Tracing Equations

In order to simplify the development of appropriate equations of ray acoustics (often called geometrical acoustics) for our present problem, we shall make the following assumptions:

(i) the speed of sound, c, and the wind velocity vector $\bar{W}$, are independent of time in the interval for which ray propagation from its origin to its destination is considered;

(ii) wave front propagation occurs over a sufficiently small region of the earth's surface for curvature of the earth to be neglected; over this region of space, the sound speed and wind velocity vector are horizontally stratified, i.e., no horizontal variation of sound speed and wind velocity vector is permitted.

The ray tracing equations for sound propagation from a supersonic aircraft are conveniently formulated in terms of three coordinate systems. The coordinate system ξ, η, z, positioned such that the wave normal lies in the ξ-z plane, is employed in the analysis of ray geometry; the x, y, z coordinate is an instantaneous coordinate for referencing the position of the aircraft while

the x_g, y_g, z is the ground-fixed observer coordinate. Now consider an aircraft propagating in the negative direction of x-axis in the x, y, z coordinate system and generating a cone of disturbance as shown in Fig. 1(a). Also shown in Fig. 1(a) is the ray cone whose tangent is inclined at the complement of the Mach angle to the aircraft flight axis. Let us suppose that a disturbance was emitted from the reference point of the aircraft (the nose) at time = 0, then after a unit time, the disturbance would have propagated to a terminal point along the ray as shown schematically in Figs. 1(b), 1(c).

With this understanding, for a particular ray under consideration the propagation equations as derived in any of Refs. 1 to 4 are briefly summarized as follows, using the symbols in Figs. 1(a), 1(b), 1(c):

$$\frac{d\xi}{dz} = (\ell c + u)\ (nc)^{-1}$$

$$\frac{d\eta}{dz} = v(nc)^{-1}$$

$$\frac{dt}{dz} = (nc)^{-1} \tag{1}$$

$$\frac{ds}{dz} = -\left[\frac{d\xi}{dz}^{2} + \frac{d\eta}{dz}^{2} + 1\right]^{1/2}$$

$$c/\ell + u = c_h/\ell_h + u_h = K, \text{ Snell's const.}$$

It should be remarked that Eq. (1) may be readily deduced from the geometry of Figs. 1(b), 1(c), by application of Snell's law for refraction of sound at an interface of discontinuity.

The following brief discussion of Figs. 1(b), 1(c) is offered in expectation that it may illuminate further understanding of the subsequent equations. In Fig. 1(b), the wind velocity vector, $\bar{W}$, is shown in its true magnitude (in accordance with the assumed horizontally stratified medium) with its components u, v. The velocity vector $\bar{s}_\ell$ represents the horizontal projection of the ray velocity vector $\bar{s}$ and is displaced from the direction of a unit normal vector $\bar{n}$ by the transverse component v, of wind velocity. The wave normal plane is inclined to the aircraft flight axis, x, at an angle γ (Fig. 1(b)) and this inclination remains constant for the ray under consideration throughout its propagation time. In Fig. 1(c), $\bar{W}_v$, $\bar{s}_v$ are the projections of the vectors $\bar{W}$, $\bar{s}$ respectively on the ξ-z plane containing the wave normal. If the wind vector is considered to be directed along the ξ-axis, then $\bar{W}_v = \bar{W}$. From Figs. 1(b), 1(c) it is readily seen that

$$\bar{s}_v = \bar{s} \cos\left\{\tan^{-1}[v(c\cos\theta + u)^{-1}]\right\} \tag{1(a)}$$

where

$$\bar{s} = c\bar{n} + \bar{W} \tag{1(b)}$$

The wavefront is defined, according to geometric acoustics, to be propagating at the undisturbed speed of sound relative to the medium through which it passes. However, relative to the ground, the velocity of the wave front in its normal

direction is augmented by the wind to yield (Fig.1(c)) 1(c)

$$\bar{N} = c\bar{n} + \bar{W} \cdot \bar{n}$$

The Snell's constant, K, shown in Fig. 1(c) is readily seen to be given by

$$K = |\bar{N}| \sec\theta$$

or

$$K = (c + \bar{W} \cdot \bar{n})\sec\theta \qquad 1(d)$$

The importance of Snell's constant in ray tracing calculations cannot be overemphasized. The ray of special interest in the present calculations is that ray which reaches the ground at grazing incidence - the ray for which the Snell's constant is a maximum (Eqn. 1(d)), at the ground. In the section that follows, we shall solve explicitly the relevant ray acoustic equations for a chosen atmospheric model.

2.2 Closed Form Solution of Ray Acoustic Equations

We observe from Fig. 1(b) that in the presence of symmetrical winds, ie., head and tail winds ($W_y = 0$), there is no lateral displacement of the ray origin from the wave normal plane. For oblique winds however, the ray is displaced from the wave normal plane in proportion to the magnitude of the transverse component, V, of wind velocity. For a constant magnitude oblique wind profile (no wind gradient) the effect of the cross wind is merely the translation of the wave normal plane a distance v.t downwind where t is the disturbance propagation time from flight altitude to the ground level. For an oblique wind decreasing to zero velocity at ground level, the ray propagates down a curved vertical surface that coincides with the wave normal plane at the ground level where v = 0.

Fortunately, the tangential component, v, of wind velocity, being perpendicular to the wave normal direction has no effect on the application of Snell's law to the wave front. Since the Snell's law applies to the effective wave normal velocity given by Eqn. 1(c) and not to the ray itself, ray tracing calculations are often performed with the wave normal parameters, the ray parameters being implicitly tied to it. Further, by suitably translating the wind profile so that W = 0 at the ground, we may obtain ray and wave front ground intersections simply by considering only the propagation in the wave normal plane as dictated by Snell's law. For corridor calculations, this is exactly what is done. Thus for the purpose of corridor calculations only, we make use of the first of Equations (1), the Snell's law, and the direction cosine relation. Eliminating the direction cosines ℓ, n between the Snell's law and the first of Equations (1) gives

$$\frac{dz}{d\xi} = \pm \left(((k-u)^2 - c^2)^{\frac{1}{2}} (c + u(k-u)c^{-1}) \right)^{-1} \qquad (2)$$

The $\pm$ sign in Equation (2) allows for upward and downward propagating sonic disturbances. Equation (2) describes the space trajectory of a sonic disturbance propagating downwards from its source in the wave normal plane. The - sign applies to the downward propagating disturbance. Note that the only requirement for Equation (2) to correctly describe the ray-ground intersection is that W = 0 at the ground - a situation which is easily created by translation. We have used the condition W = 0 instead of v = 0 (W does not have to be zero for v to be zero) because the whole analysis is carried out in terms of wind magnitude rather than its components. Thus the use of W = 0 rather than v = 0 is for symbolic convenience rather than a limitation. It must be pointed out that

for consideration of the geometry of the ray through the atmosphere, ray propagation in η-direction must be taken into account. For completeness of this study, the expression for ray propagation in η-direction is derived herein at the appropriate point.

If the speed of sound c and wind velocity component u along the wave normal plane are known as functions of z, then Equation (2) may be evaluated directly by quadrature. However, it is instructive to assume a piecewise linear atmosphere and seek a closed form solution of Equation (2). Accordingly, define the model wind profile

$$u' = ug + \alpha z$$

and sound speed profile

$$c = cg - \beta z$$

so that relative to the wind speed at the ground ug, we have

$$u = \alpha z$$

$$c = cg - \beta z \tag{3}$$

where α, β are positive quasi-constants (piecewise constant wind and sound speed gradients respectively). Upon substituting Equation (3) into Equation (2) we obtain the ray slope equation as

$$\frac{dz}{d\xi} = - \frac{[(k-\alpha a)^2-(6g-\beta z)^2]^{\frac{1}{2}}(\hat{c}g-\beta z)}{(cg-\beta z)^2+ \alpha z(k-\alpha z)} \tag{4}$$

Let

$$z_1 = k - \alpha z$$

$$z_2 = c_g - \beta z \tag{5}$$

$$z_r = z_1/z_2$$

Then

$$\frac{dz}{d\xi} = - \frac{s_o(z_r^{\ 2} - 1)^{\frac{1}{2}}}{\alpha c_g z_r^{\ 2} - \alpha k z_r + s_o} \tag{6}$$

where

$$s_o = \beta k - \alpha c_g$$

Combining the parameters in Equation (5) and differentiating, gives:

$$dz = \frac{s_o dz_r}{(\beta z_r - \alpha)^2}$$

and using this in Equation (6) gives:

$$\frac{dz_r}{d\xi} = -\frac{[z_r^2-1]^{\frac{1}{2}}(\beta z_r-\alpha)^2}{\alpha c_g z_r^2-\alpha k z_r + s_o} \tag{7}$$

The steps leading to Equations (5) - (7) are necessary to bring Equation (4) to a form permitting the use of standard integrals. Intergrating Equation (7) we have

$$\xi - \xi_o = -\int_{z_{r_1}}^{z_{r_2}} \frac{\alpha c_g z_r - \alpha k z_r + s_o}{(\beta z_r-\alpha)^2(z_r^2-1)^{1/2}}\, dz_r \tag{8}$$

which upon employing partial fractions integrates to

$$\xi - \xi_o = -\left\{(\beta k-\alpha c_g)I_4 + \alpha/\beta[\alpha I_4 + I_3]\left(\frac{\alpha c_g}{\beta} - k\right)\right\}_{z_{r_1}}^{z_{r_2}} \tag{9}$$

where

$$I_3 = \frac{1}{(\alpha^2-\beta^2)^{\frac{1}{2}}} \mathrm{TAN}^{-1}\left[\frac{(\beta^2-\alpha^2)-\alpha(\beta z_r-\alpha)}{\beta[(\alpha^2-\beta^2)(z_r^2-1)]^{\frac{1}{2}}}\right] \tag{10}$$

$$I_4 = \frac{1}{(\beta^2-\alpha^2)}\left[\alpha I_3 - \frac{\beta(z_r^2-1)^{\frac{1}{2}}}{(\beta z_r-\alpha)}\right] \tag{11}$$

$$z_r = \frac{k-\alpha z}{c_g-\beta z} \tag{12}$$

After considerable algebraic manipulations, Equation (9) reduces to

$$\xi_{i+1} = \xi_i + \left\{\left[(k-u_{i+1})^2-c_{i+1}^2\right]^{\frac{1}{2}} - \left[(k-u_i)^2-c_i^2\right]^{\frac{1}{2}}\right\}$$

$$\times\ (z_i-z_{i+1})\ (c_{i+1}-c_i)^{-1} \tag{13}$$

Strictly speaking, Equation (13) represents the true ray trajectory in the wave-normal plane only for symmetrical winds. For oblique winds decreasing to zero velocity at the ground, Equation (13) represents the projection of ray trajectory on the wave normal plane until the ground is reached at which point it represents the true ray trajectory.

Noting that Equation (13) becomes singular when $c_{i+1} = c_i$, we seek a solution valid under this condition. Physically, this condition implies that sonic boom can spread to infinite distances in an isothermal atmosphere. The solution valid under isothermal conditions in the presence of winds is

$$\xi = \frac{1}{2} \sum_{i=1}^{N-1} \left\{ c_{i+1}^2 \ln\left[(k-u_{i+1}) + \left[(k-u_{i+1})^2 - c_{i+1}^2\right]^{\frac{1}{2}}\right] + k\left[(k-u_{i+1})^2 - c_{i+1}^2\right]^{\frac{1}{2}} \right.$$

$$\left. - c_{i+1}^2 \ln\left[(k-u_i) + \left[(k-u_i)^2 - c_{i+1}^2\right]^{\frac{1}{2}}\right] - k\left[(k-u_i)^2 - c_{i+1}^2\right]^{\frac{1}{2}} \right\}$$

$$\times (z_{i+1} - z_i)\, c_{i+1}^{-1}\, (u_{i+1} - u_i)^{-1} \tag{14}$$

N = number of atmospheric layers for which data is given.

Under isothermal conditions and no winds, the valid solution derived from Equation (13) is:

$$\xi = c_{i+1}\,(z_i - z_{i+1})\,[k^2 - c_{i+1}^2]^{-1/2} \tag{15}$$

As discussed in Section 2.1, for symmetrical winds the ray is contained in the wave normal plane, hence the equations developed up to this point are sufficient to carry out ray tracing and corridor width calculations. However, for completeness of this study, and to enable the reader perform ray tracing in the presence of cross-winds, the ray propagation in η-direction must be included. To do this, we eliminate the direction cosines ℓ, n between the second of Equations (1) and the Snell's law, to obtain

$$\frac{d\eta}{dz} = - vc^{-1}\,(k-u)\,[(k-u)^2 - c^2]^{-1/2} \tag{16}$$

The velocity components u, v are related (see Fig. 1(b)) through the expressions

$$u = - W\cos(H-D-\gamma)$$

$$v = W\sin(H-D-\gamma) \tag{17}$$

so that

$$v = -u\tan(H-D-\gamma) \tag{18}$$

Substituting Equation (18) in (16) and going through the same procedures as before, one obtains

$$\frac{d\eta}{dz} = \frac{\alpha z \tan(H-D-\gamma)}{(cg-\beta z)} \; \frac{(k-\alpha z)}{[(k-u)^2 - (cg-\beta z)^2]^{1/2}} \tag{19}$$

which after substitutions and use of partial fractions integrates to

$$\eta = \sum_{i=1}^{N-1} \mu_i(\mu_i c_g - K) \left\{ \left[tn_{i+1} . TAN^{-1} \left[\frac{c_{i+1} - \mu_i (K-u_{i+1})}{[(K-u_{i+1})^2 - c_{i+1}^2]^{1/2}} \right] - \mu_i \frac{[(K-u_{i+1})^2 - c_{i+1}^2]^{1/2}}{(K-u_{i+1} - \mu_i c_{i+1})} \right. \right.$$

$$\left. \left. - \left[tn_i . TAN^{-1} \left[\frac{c_i - \mu_i (K-u_i)}{[(K-u_i)^2 - c_i^2]^{1/2}} \right] - \mu_i \frac{[(K-u_i)^2 - c_i^2]^{1/2}}{(K-u_i - \mu_i c_i)} \right] \right\}$$

$$x\ (Z_{i+1} - Z_i).(c_{i+1} - c_i)^{-1} \tag{20}$$

where

$$\mu_i = (u_{i+1} - u_i)\ (c_{i+1} - c_i)^{-1}$$

$$tn_{i+1} = \tan\ (H - D_{i+1} - \gamma) \tag{21}$$

Under isothermal conditions, $\mu_i \to \infty$ and Equation (20), like Equation (13), is invalid. The valid solution is then

$$\eta = -1/2 \sum_{i=1}^{N-1} tn_{i+1} \left\{ (k+u_{i+1})[(k-u_{i+1})^2 - c_{i+1}^2]^{1/2} - c_{i+1}^2 \ell n \left[(k-u_{i+1}) \right. \right.$$

$$\left. \left. +[(k-u_{i+1})^2 - c_{i+1}^2]^{1/2} \right] \right\} - tn_i \left\{ (k-u_i)[(k-u_i)^2 - c_i^2]^{1/2} \right.$$

$$\left. - c_{i+1}^2 \ell n \left[(k-u_i) + [(k-u_i)^2 - c_{i+1}^2]^{1/2} \right] \right\}$$

$$x\ (z_{i+1} - z_i)\ (u_{i+1} - u_i)^{-1} \tag{22}$$

The lateral spread of sonic boom is obtained by coordinate transformation -

$$\begin{aligned} x &= \xi\cos\gamma - \eta\sin\gamma \\ y &= \xi\sin\gamma + \eta\cos\gamma \end{aligned} \tag{23}$$

The maximum lateral spread of the sonic boom is given by the so called 'cut-off' point. This point is determined by the last ray to strike the ground at grazing incidence. The necessary condition and equation for fulfilment of grazing incidence will now be considered.

2.3 Geometrical Considerations

The purpose of this section is to express the wind components along and perpendicular to the wave normal plane in terms of the wind magnitude and direction, and to define the wave normal plane for the condition of grazing incidence for the ray under consideration. The angle defining the wave normal plane is needed to transform the wave normal coordinate system to the aircraft coordinate system.

Consider the aircraft to be moving in the negative direction of x-axis as shown in Fig. 1(a). Then, relative to the coordinate system moving with the wind at the aircraft altitude, the following geometrical relations are derived from Fig. 1(a):

$$\sin\gamma = \lambda\sin\phi\,[1 + \lambda^2\sin^2\phi]^{-1/2}$$

$$\cos\theta = -M^{-1}\,[1 + \lambda^2\sin^2\phi\,]^{-1/2}$$

also

$$\cos\theta = -(M\cos\gamma)^{-1} \tag{24}$$

where

$$\lambda = (M^2-1)^{-1/2}$$

It is convenient to perform the calculations in a coordinate system moving with the wind because it lends itself to the use of simple geometrical relations such as given in Equations (24). If calculations are performed in the coordinate system fixed to the earth, then the deformation of the initial wave system due to the wind at the flight altitude will have to be considered. In the coordinate system moving with the wind at the flight altitude, the wind velocity is zero at the flight altitude. The final result required in a ground fixed coordinate system is obtained through a galilean transformation.

We shall specify the wind direction such that the heading angle, H, is measured clockwise positive from the true North and winds reported as blowing from the direction D, also measured positive clockwise from the North - Fig. 1(b).

The components of wind velocity along and perpendicular to the flight path are:

$$Wx = -W\cos(H-D)$$

$$Wy = W\sin(H-D) \tag{25}$$

hence the component of wind velocity parallel to wave normal plane is

$$u(z,\gamma) = Wx\cos\gamma + Wy\sin\gamma$$

$$= -W\cos(H-D-\gamma) \tag{26}$$

and perpendicular to wave normal plane is

$$v(z,\gamma) = -Wx\sin\gamma + Wy\cos\gamma$$

$$= W\sin(H-D-\gamma) \tag{27}$$

The angle, γ, defining the particular wave normal plane for grazing incidence at the ground may now be determined. Rewrite Snell's law given in Equation (1) as:

$$K' = c_h/\cos\theta_h + u_h = cg/\cos\theta_g + ug \tag{28}$$

The condition for grazing incidence at the ground requires that $\cos\theta_g = -1$, so that from Equation (28), we have

$$K' = -cg + ug \tag{29}$$

Then relative to the coordinate system moving with the wind at the aircraft altitude, we have

$$k = -cg - u_h + ug \tag{30}$$

and

$$\cos\theta_h = c_h/k \tag{31}$$

Note that by virtue of the use of a moving coordinate system, the ray and the wave normal are coincident at the flight level where $u_h = 0$, hence the use of initial wave normal slope in this calculation. From geometrical considerations (Fig. 1(b))

$$u(z, \gamma) = -W\cos(H-D-\gamma)$$

hence

$$\cos\theta_h = c_h[-cg + (W_g-W_h)\cos(H-D-\gamma)]^{-1} \tag{32}$$

But from Equation (24), $\cos\theta_h = -(M\cos\gamma)^{-1}$ and using this in Equation (32) gives

$$\cos\gamma = \frac{c_g(c_nM-W\cos\omega) \pm[W\sin\omega \quad (c_nM)^2+W^2-2c_hMW\cos\omega - c_g{}^2]^{\frac{1}{2}}}{(c_hM)^2 - 2c_nMW\cos\omega + W^2} \tag{33}$$

where $\omega = H-D$. The - sign before the radical applies to the leeward side and the + sign to the windward side of the flight track.

The angle, γ, defining the wave normal plane is given by Equation (33) while the initial wave normal slope at emission altitude is given by Equation (32). Equation (33) is important and leads to some interesting results. It is possible to determine the magnitude of wind velocity required to confine sonic boom to the flight path alone.

2.4 Winds Required for 'Cut-Off' of Sonic Boom

Under the flight path, $\gamma = 0$, $\theta = \mu'$, the complement of the Mach angle relative to wind. If the heading angle is taken arbitrarily as 180° (Fig. 1(b)) then for head wind, $D = 180^\circ$ so that $H-D = 0$. Substituting these parameters in Equation (33) gives the limiting head wind as:

$$W = c_hM - cg \tag{34}$$

Similarly for tail wind,

$$W = cg - c_h M \tag{35}$$

2.5 Ray Propagation Time History

The ray propagation time is given in Equation (1) as:

$$\frac{dt}{dz} = (nc)^{-1} \tag{36}$$

where

$$nc = c[(k-u)^2-c^2]^{\frac{1}{2}} (k-u)^{-1} \tag{37}$$

Substituting Equation (37) in (36) and integrating between any two points, gives the ray propagation time between these two points as:

$$t = \int_{z_1}^{z_2} (k-u) [(k-u)^2-c^2]^{-\frac{1}{2}} c^{-1} dz \tag{38}$$

Making use of Equation (3) in (38) gives:

$$t = \int_{z_1}^{z_2} (k-\alpha z)(c_g-\beta z)^{-1}[(k-\alpha z)^2-(cg-\beta z)^2]^{-\frac{1}{2}} dz \tag{39}$$

If we go through the same arguments and substitutions as in Section 2.1, we obtain from Equation (39), the following result:

$$t_h-t = \int_{z_{r_h}}^{z_r} z_r(\beta z_r-\alpha)^{-1}[z_r^2-1]^{-\frac{1}{2}} dz_r \tag{40}$$

where t_h is the time of emission of the ray at the flight altitude, and z_r is defined in Equation (5).

By writing

$$z_r(z_r-\alpha/\beta)^{-1}\beta^{-1}[z_r^2-1]^{-\frac{1}{2}} \equiv \beta^{-1} [z_r^2-1]^{-\frac{1}{2}} \left\{1+ \alpha/\beta(z_r-\alpha/\beta)^{-1}\right\}$$

and substituting in Equation (40), one obtains

$$t_h-t = \left[\beta^{-1}\cosh^{-1}z_r+ \alpha\beta^{-2}\left[[1-(\alpha/\beta)^2]^{-\frac{1}{2}}\mathrm{TAN}^{-1}\left\{\frac{(1-(\alpha/\beta)^2)-\alpha/\beta(z_r-\alpha/\beta)}{[1-(\alpha/\beta)^2)(z_r^2-1)]^{-\frac{1}{2}}}\right\}\right]\right]_{z_{r_h}}^{z_r} \tag{41}$$

Let $\mu = \alpha/\beta$ as defined in Equation (21), then Equation (41) becomes

$$t_h - t = \beta^{-1}\left\{\cosh^{-1}\left(\frac{K-u}{c}\right) + \mu\,\mathrm{TAN}^{-1}\left[\frac{c - \mu\,(K-u)}{[(K-u)^2 - c^2]^{1/2}}\right]\right\}_{z_h}^{z} \tag{42}$$

where it is understood that in Equation (42) $u = u(z)$, $c = c(z)$. Recasting Equation (42) into a form more suited to digital operations, one obtains:

$$t = \sum_{i=1}^{N-1} \beta_i^{-1}\left\{\cosh^{-1}\left(\frac{K-u_{i+1}}{c_{i+1}}\right) + \mu_i \mathrm{TAN}^{-1}\left[\frac{c_{i+1}-\mu_i(K-u_{i+1})}{[(K-u_{i+1})^2 - c_{i+1}^2]^{1/2}}\right]\right.$$

$$\left. - \cosh^{-1}\left(\frac{K-u_i}{c_i}\right) - \mu_i \mathrm{TAN}^{-1}\left[\frac{c_i-\mu_i(K-u_i)}{[(K-u_i)^2 - c_i^2]^{1/2}}\right]\right\} \tag{43}$$

Note that Equation (43) represents a general equation for calculation of propagation time for any ray which may or may not reach the ground. For a quiescent state between layers, Equation (43) reduces to

$$t = \sum_{i=1}^{N-1}\left\{\cosh^{-1}\frac{k}{c_{i+1}} - \cosh^{-1}\frac{k}{c_i}\right\}(z_{i+1}-z_i)(c_{i+1}-c_i)^{-1} \tag{44}$$

For a special case in which a ray is required to reach the ground at grazing incidence, Equation (44) takes the form

$$t = \beta^{-1}\cosh^{-1}(cg/c_h) \tag{45}$$

where β is the sound speed gradient.

This shows that for this particular ray, the propagation time from the flight altitude to the ground depends only on the altitude, for a given ground condition.

The general solution applicable to isothermal state between layers with winds is given by

$$t = \sum_{i=1}^{N-1}(c_{i+1}\alpha_i)^{-1}\left\{[k-u_i)^2 - c_{i+1}^2 - c_{i+1}^2]^{1/2} - [(k-u_{i+1})^2 - c_{i+1}^2]^{1/2}\right\} \tag{46}$$

For an isothermal, quiescent state between layers, the ray propagation time is simply

$$t = k\,c_{i+1}^{-1}(k^2 - c_{i+1}^2)^{-1/2}(z_{i+1} - z_i) \tag{47}$$

The transformations between a ground fixed coordinate system and that defined by Equation (23) are

$$\bar{x}_g = \bar{x} - W_h \cos(H-D-\cdots)\, t_g \qquad (48)$$
$$\bar{y}_g = \bar{y} + W_h \sin(H-D-\cdots)\, t_g$$

2.6 Shock-Ground Intersection

Equation (48) gives the ray-ground intersection when $z = 0$. We must clearly distinguish the difference between the ray and the shock-ground intersections. By ray-ground intersection, we mean the ground locus of acoustic disturbances emitted from the aircraft simultaneously. These disturbances, however, arrive at the ground at different times, t_g, depending on their initial direction of propagation at emission time, t_B. The shock-ground intersection on the other hand is the locus of sonic disturbances reaching the ground simultaneously - as sonic boom. These disturbances may have been emitted at different times.

For steady, level flight being treated herein, the coordinates of shock-ground intersection are

$$x_s = x - W_h \cos(H-D-\cdots)t_m - c_h M(t_g - t_m) \qquad (49)$$
$$y_s = y + W_h \sin(H-D-\cdots)t_m$$

where t_g = time of arrival of disturbance at the ground along the ray

t_m = time of arrival at the ground, of the disturbance propagating directly under the flight path (minimum ray propagation time).

We shall in the next section discuss the results obtained from computer solutions of the equations developed in this section.

3.0 DISCUSSION OF RESULTS

3.1 Standard Atmosphere

In the last section we discussed the significance of the analytical expressions on a qualitative basis. In this section, we shall discuss the computational results on a qualitative basis. It is reasonable to base our discussions on flight altitudes within the stratosphere, since the proposed flight altitudes of the SST's fall within this domain. Now, for a given atmospheric model, the main parameters among others, that influence the sonic boom corridor are the flight altitude and Mach number, wind and temperature gradients and wind direction. It is therefore natural to determine the variability of sonic boom corridor in terms of these physical, measurable parameters.

Equations (34), (35), displayed graphically in Fig. 2 show that for flights above the tropopause in a standard, quiescent atmosphere, sonic booms do not reach the ground for $M = C_g/C_h < 1.15$. This Mach number apparently corresponds to the so-called 'cut-off' Mach number in a quiescent atmosphere. The term 'so-called' is used here because booms are still heard on the ground until well past the 'cut-off' condition. At this condition for which the wave cusp just reaches the ground one has a superboom (focussed boom) on the ground. As we move further into the 'cut-off' region, the cusp moves higher above the

ground and the boom degenerates into a mere rumble. Beyond the 'cut-off' point, the effect of the acoustic disturbances is negligible.

A further interesting characteristic of the tail wind is clearly evident in Fig. 2. Observe that whereas 'cut-off' Mach number for flights above the tropopause in a quiescent atmosphere is 1.15, tail winds ranging from zero to 87.6 knots at the flight level can reduce the 'cut-off' Mach number from 1.15 to 1.0. Further remarks regarding the interpretation of Fig. 2 will be made when discussing the effects of altitudes on sonic boom corridor.

The effects of flight Mach number and wind speed on sonic boom corridors are shown in Fig. 3. In this figure, five groups of corridors (length of each strip represents corridor widths; width of each strip is non-dimensional) labelled, A, B, C, D, E, are shown. Within each group, starting from the left and proceeding to the right, the strips represent no wind, tail wind, side wind (blowing from right of flight path to the left), and head wind respectively. The coordinate position of each group of corridors in relation to the zero corridor lines - PQ, PR, is similarly labelled in Fig. 2, in a wind-Mach number coordinate system. Since line PR, Fig. 2, represents wind and Mach number combinations for which the corridor is reduced to a line along the flight track for head winds, it is to be expected that as we move away from PR along a line of constant wind (as points A, B, C) the corridor width should increase, as indeed it does in Fig.3. An alternative description of the line PR in Fig. 2 is that it represents wind speed and aircraft Mach number combination for which only the ray directly under the flight path just reaches the ground tangentially. Thus for an aircraft in a straight, level flight at constant speed, the rays emitted at successive times will form an envelope. PR and PQ are thus superboom paths for head and tail winds respectively under the aircraft.

The author shares the opinions expressed in Refs. 3, 5 that the ratio 'corridor with winds/corridor without winds' remains essentially constant for aircraft speeds larger than $M = 1.3$. However, the author wishes to stress that the opinions expressed in these references - that the ratio 'corridor with winds/corridor without winds' is close to unity for $M > 1.3$ is valid only for small and medium wind profiles as used in those references and shown in Fig. 10.

It happens that the magnitude of the wind speed at the flight altitude has a predominating influence on the corridor width, over the effects of actual wind distribution, just as the effect ground temperature predominates that of temperature distribution. Now, for the 'mean zonal' wind profile used in Ref. 3, at 50,000 ft., the wind speed is about 31.4 knots and at the ground is 5.9 knots. Thus, relative to wind at the ground fixed coordinate, the wind at flight altitude is 25.5 knots, which is small compared with the speed of sound at the flight altitude. In the presence of strong winds, such as 200 knots, the ratio 'corridor with winds/corridor without winds' is neither constant nor close to unity as is shown by corridor groups D and E in Fig. 3.

Further effects of wind magnitude and direction are shown in Fig. 4 for constant altitude - 40,000 ft. and $M = 1.6$ for linear wind profiles decreasing to zero at the ground. Observe that for a 20-knot wind, the increase in corridor width by a tail wind (increase above no wind corridor) is about the same as the decrease by a head wind - the type of effect expected from a linear theory solution. However, as the wind speed increases, the departure from the above trend becomes more pronounced. The effect of side (cross) winds ($D = 90^{\circ}$) is clearly evident in Fig. 4. The general effect of the side winds is the displacement of the boom corridor to the lee of the flight track. The particular effect is a

slight reduction in corridor width for strong winds ($W > 60$ knots, say). Observe in Fig. 4 that for a side wind of 20 knots, the corridor width is unaltered from the no wind case. For $W = 64.5$ knots, the corridor width is reduced by 2 miles from the no wind case, and for $W = 200$ knots, by 5 miles or 13%. Thus for small and moderate side wind profiles, the boom corridor width is, for all practical purposes, the same as for no wind.

The effect of flight altitudes on sonic boom corridor is calculated and shown in Fig. 5 for $M = 1.6$, $W = 64.5$ knots at the flight altitude and decreasing linearly to zero at the ground. This figure, which is self-explanatory, in effect illustrates the effect of wind gradients. High wind gradient reduces boom corridor widths, just as high temperature gradient does. We can infer from Fig. 5 that if a 64.5-knot wind at 40,000 ft. creates a narrower corridor on the ground than the same wind at 50,000 ft., then a wind that causes a ray emanating from 50,000 ft. to reach the ground only under the flight track at grazing incidence (a point on PR, Fig. 2) will cause complete cutoff for the ray emanating from 40,000 ft. With this explanation in mind, let us now go back to Fig. 2. The inference made from Fig. 5 implies that although Fig. 2 is valid for flights between 36,080 ft. and 65,000 ft., the actual altitude of origin of the disturbance must be known before a valid interpretation of Fig. 2 can be made for any case of interest. This clarification has not been made in any literature where Fig. 2 has been presented, and the author feels that the absence of this clarification could lead the reader to wrong conclusions.

The present calculation is compared with the results of Ref. 3 for the mean zonal wind profile shown in Fig. 10 in a standard atmosphere, at 50,000 ft., $M = 1.2$. Agreement of both results is excellent, despite the fact that Ref. 3 did not assume linear wind and temperature profiles.

3.2 Non-Standard Atmospheres

In general, ray propagation characteristics depend on the local variations of atmospheric properties; however, whether a ray gets to the ground or not, depends mainly on the distributions of wind and temperature at the flight altitude and the ground. It is inferred from Equation 1(d) that the wave normal which determines the maximum lateral spread of sonic boom is the one for which the Snell's constant is a maximum. Relative to the coordinate system moving with the wind at the flight altitude, Equation (32) indicates that the initial inclination to the horizontal of the wave normal that is to reach the ground at grazing incidence, is dictated solely by the wind and the sound speeds at the flight level and the ground. Thus for large temperature difference, ΔT, between the ground and the flight altitude, the ray leaves its source at a steep inclination to the horizontal and hence can withstand temperature inversions typical of non-standard atmospheres. On the other hand, for small ΔT, the ray leaves its source at a shallow inclination to the horizontal, thus making it very sensitive to temperature inversions. Under this condition, a slight abrupt change in atmospheric properties, such as temperature inversions, results in very large values for sonic boom corridors.

For the purpose of comparison, sonic boom corridors have been computed for a typical Canadian town, Maniwaki, Quebec, for the month of January, using the mean meteorological data provided in Refs. 9 and 10 and plotted in Figs. 8 and 9. Comparison is made in Fig. 7, with the corridors labelled group C in Fig. 3. Observe that with ΔT (winter) just about 60% of ΔT (standard atm.), winter corridor is more than triple the standard atmosphere corridor for tail

wind, other meteorological parameters held constant. It is interesting to note that the temperature at the flight altitude - 50,000 ft, remains about the same for winter and standard atmospheres (see Fig. 8), so that the ground temperature is mainly responsible for the difference in corridors. This leads us to conclude that the ground surface temperature must be the most important meteorological factor affecting the sonic boom corridor.

4.0 CONCLUSIONS

Calculation of sonic boom corridors based on closed form solutions of the ray acoustic equations using piecewise linear atmospheric models of winds and temperatures has been accomplished. The computer program for these calculations processes 5 different cases, each of 8 atmospheric layers in 3 minutes using the comparatively slow computer - IBM 1130.

The results show that whereas a complete cut-off of sonic boom for flights above the tropopause in a quiescent standard atmosphere occurs for $M < 1.15$, the cut-off Mach number is reduced by the tail wind. Specifically, a tail wind of about 87 knots will reduce the cut-off Mach number to $M = 1.0$ at the appropriate altitude within the tropopause.

Theoretically, sufficiently high head winds will confine the sonic to the flight track only, with attendant focussing effects; higher winds will cause boom cut-off. However, the required head winds increase with Mach number and exceed 200 knots above $M = 1.5$. The main effect of side winds is to shift the corridor laterally leeward with respect to the flight track, the shift being in proportion to the wind strength. Side winds less than 20 knots at 40,000 ft. do not alter the width of the corridor from the no wind case, but higher winds cause a slight reduction. In particular, a side wind of 200 knots at 40,000 ft. causes a 13% reduction in corridor width from no wind case.

It is found that for small to moderate wind profiles and $M > 1.5$, the increase in corridor width (above no wind case) due to tail wind is approximately equal to the decrease due to head wind. For stronger winds at the same Mach number, head winds produce progressively higher decrease in corridor width than the increase due to tail winds. The largest variations in corridor widths due to tail winds occur for $M < 1.3$.

The effect of winds on sonic boom corridor is more pronounced for flights above the tropopause where isothermal conditions prevail, but is less significant for flights below the tropopause where temperature effects are dominant. Based on the results for non-standard atmospheres, the ground temperature is the greatest single meteorological parameter affecting the sonic boom corridor; the influence of ground temperature is such that higher than standard temperature constricts it whilst lower than standard temperature expands it.

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APPENDIX

Snell's Law and Ray Tracing

The purpose of this appendix is to define Snell's law in the context used for geometric acoustic propagation in a stratified atmosphere, and to explain its role in ray tracing.

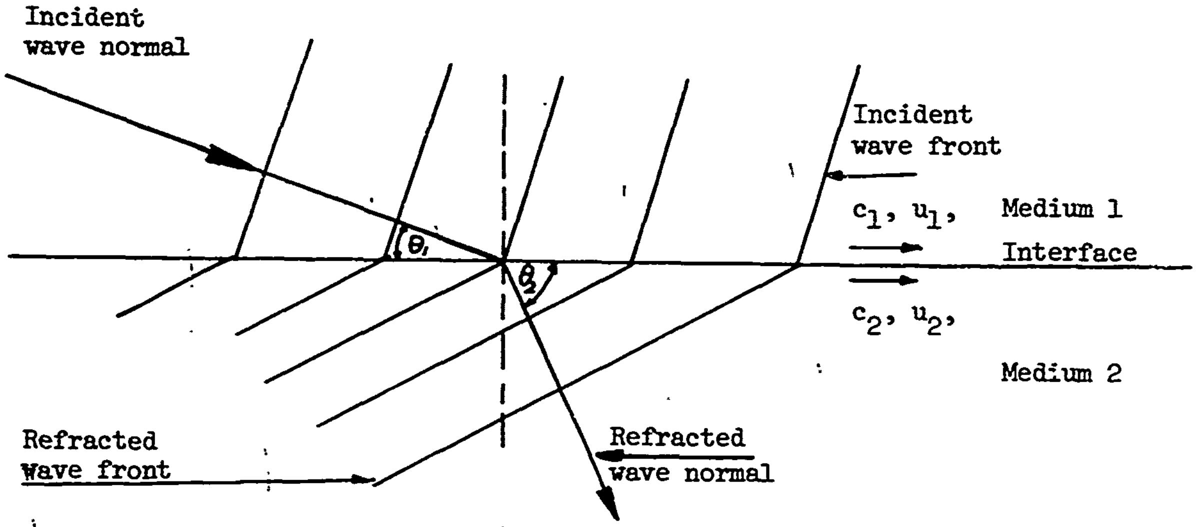
In geometric acoustics, disturbances are propagated on the wave front along the rays. The wave front propagates such that its normal velocity relative to the medium is the undisturbed speed of sound.

In geometric optics, Snell's law of refraction defines the refractive index of a medium in terms of the speed of propagation of light wave through it, i.e.,

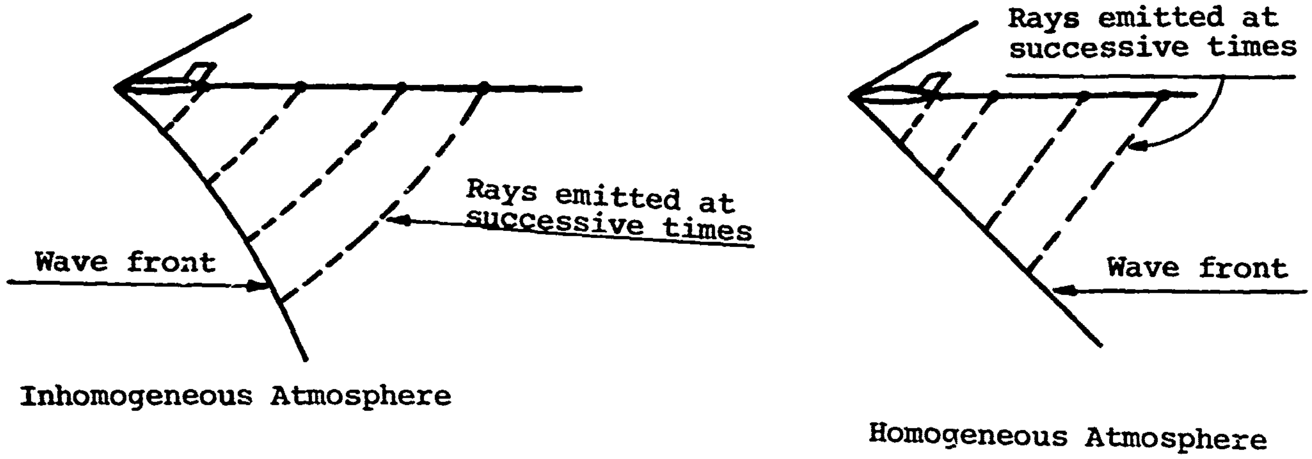
$$\text{Refractive index} = \frac{\text{speed of light in vacuum}}{\text{speed of light in medium}}$$

For the propagation of a sound wave in a horizontally stratified, quiescent atmosphere, an analogous refraction law states that the trace velocity of the wave front along the interface separating two layers is conserved. Even when there is relative motion between layers (see sketch below) the conservation of the trace velocity of the wave front still holds; thus the equivalent of Snell's law for a stratfied moving medium may be states mathematically as

$$\frac{c_1}{\cos\theta_1} + u_1 = \frac{c_2}{\cos\theta_2} + u_2 = \text{constant, } k$$



In keeping with the principle of geometric acoustics, it is not the actual ray path - a kinematic entity, which is used for ray tracing, but its geometric equivalent - its projection on the wave-normal plane. Hence to calculate the ray, the information provided by Snell's law about the wave normal slope at each interface is used to evaluate the slope of the geometric ray at the interface.



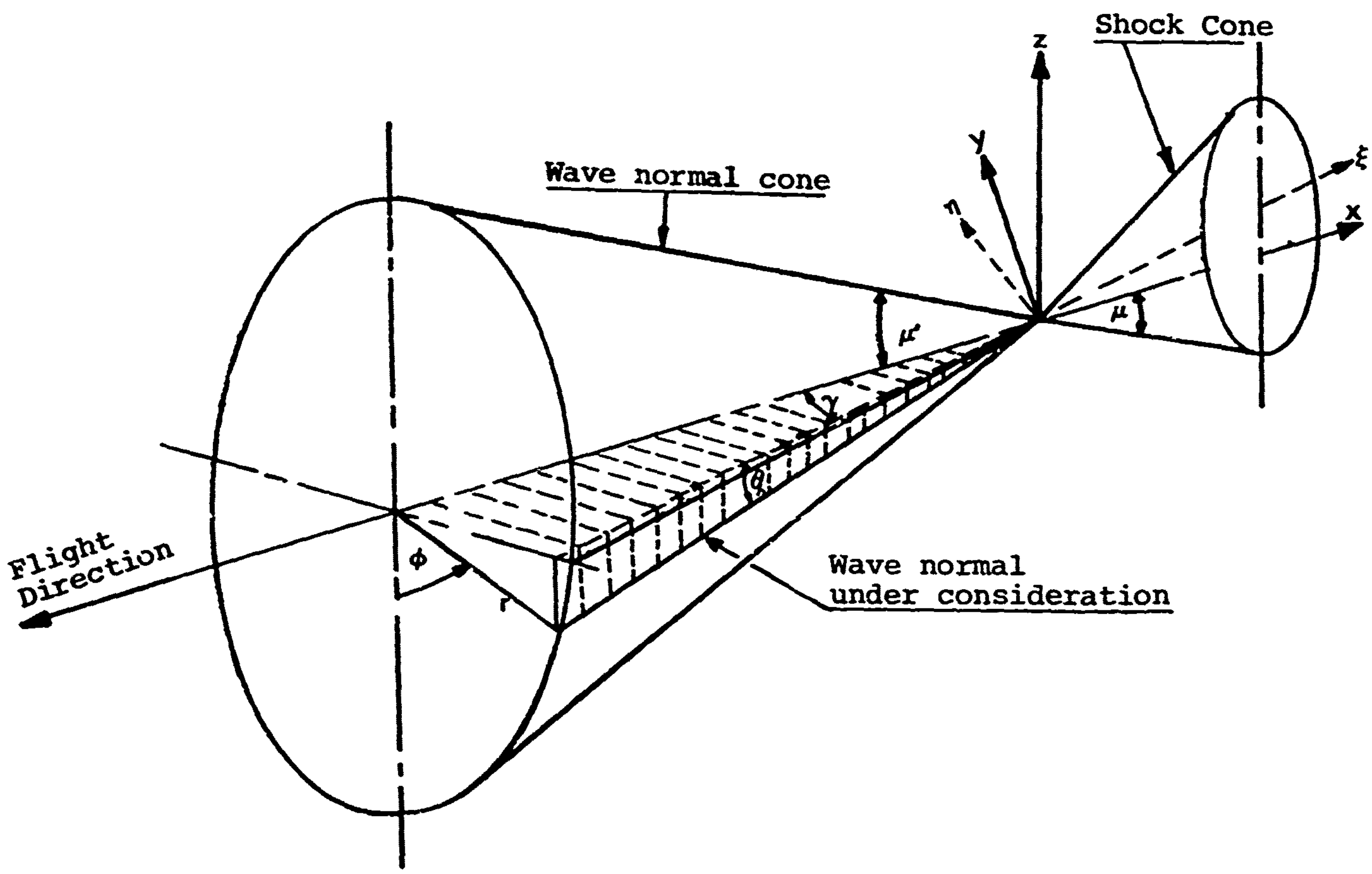


FIG. 1(a). SKETCH SHOWING COORDINATE ORIENTATION AND INITIAL WAVE NORMAL DIRECTION RELATIVE TO WIND AT FLIGHT ALTITUDE.

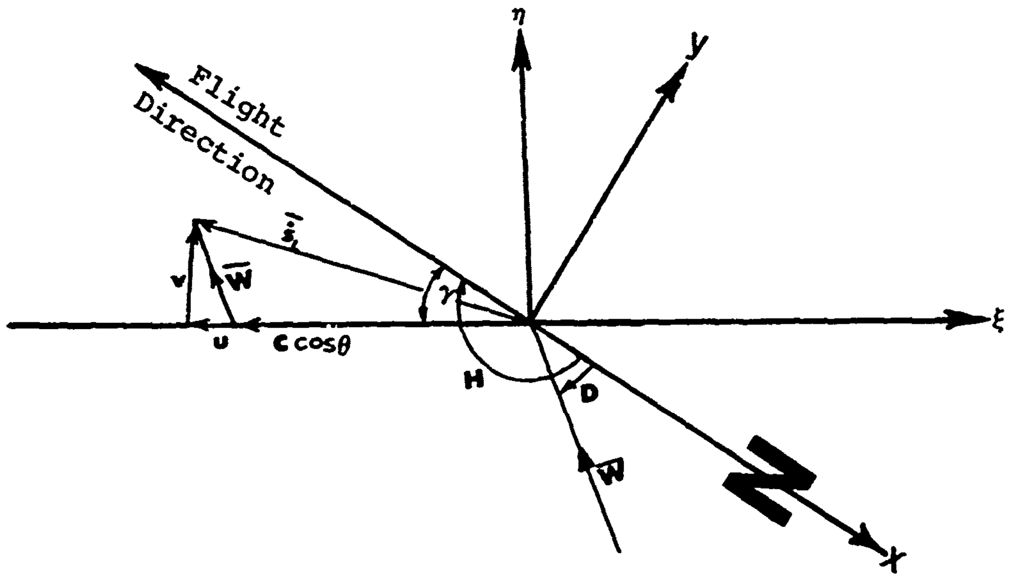


FIG. 1(b) SKETCH SHOWING THE PROJECTIONS OF THE RAY AND THE WAVE NORMAL VELOCITIES ON THE HORIZONTAL PLANE.

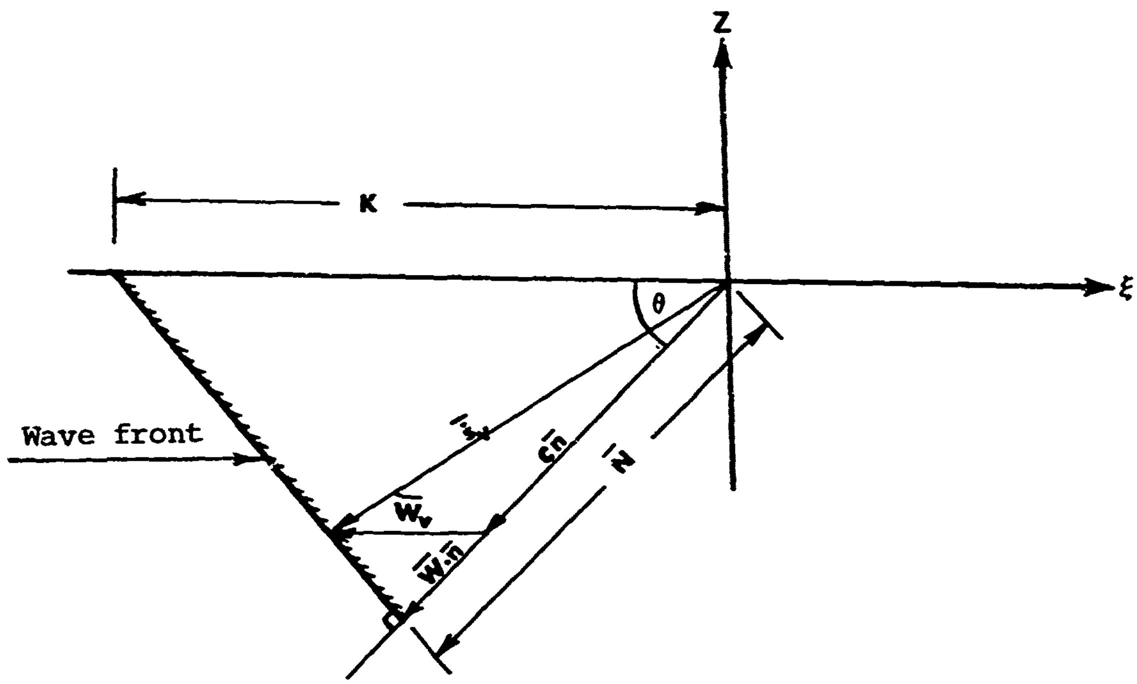


FIG. 1(c). VELOCITY PLOT AND WAVE FRONT ORIENTATION IN THE ξ-Z PLANE.

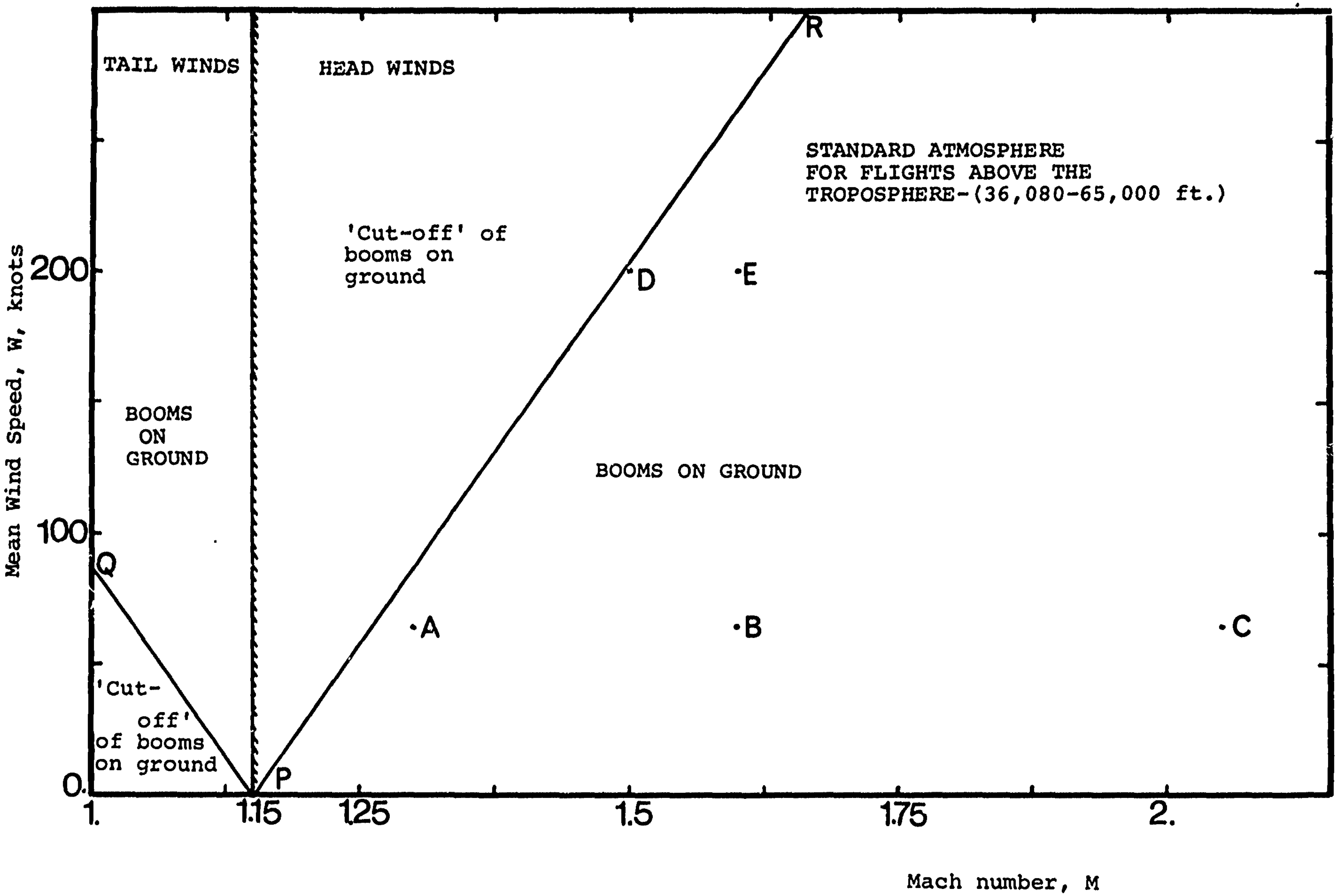


FIG. 2. WINDS REQUIRED FOR 'CUT-OFF' OF SONIC BOOM.

STANDARD ATMOSPHERE
ALTITUDE = 50,000 ft.
A, M = 1.3, W = 64.5 knots
B, M = 1.6, W = 64.5 knots
C, M = 2.05, W = 64.5 knots
D, M = 1.5, W = 200 knots
E, M = 1.6, W = 200 knots

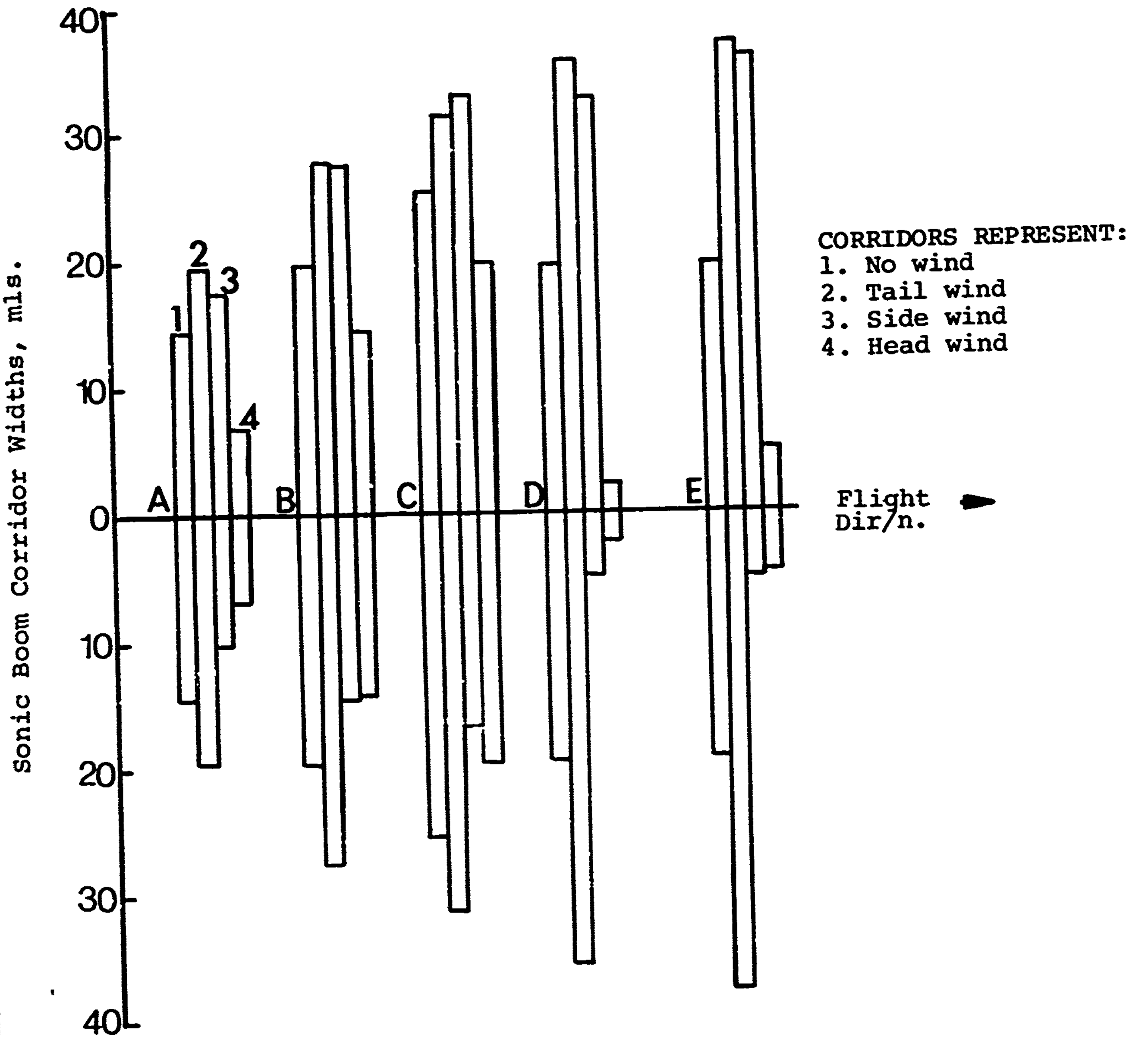


FIG. 3. EFFECTS OF MACH NUMBER AND WIND MAGNITUDE ON SONIC BOOM CORRIDOR.

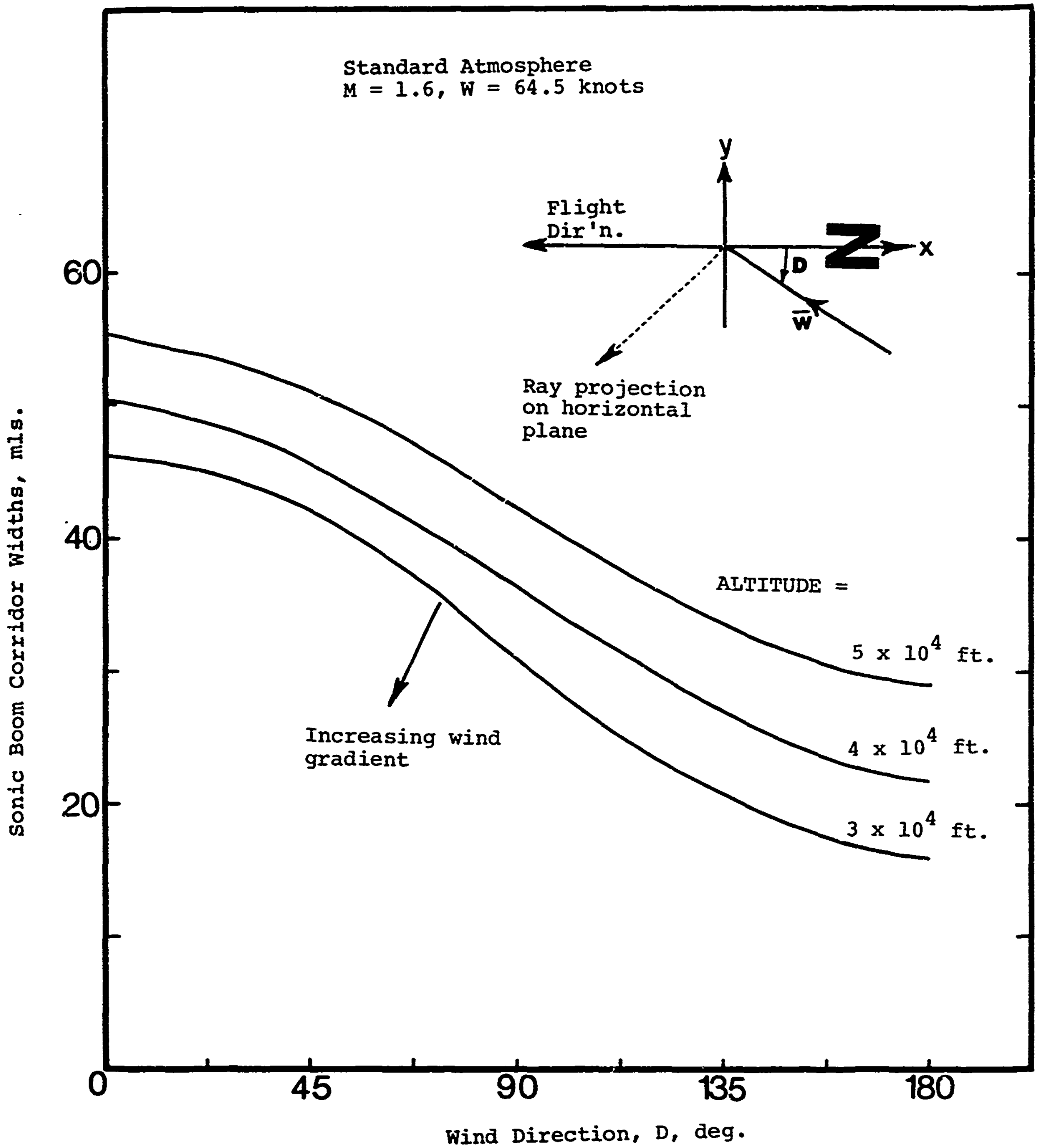


FIG. 4. EFFECTS OF ALTITUDE AND WIND DIRECTION ON SONIC BOOM CORRIDOR.

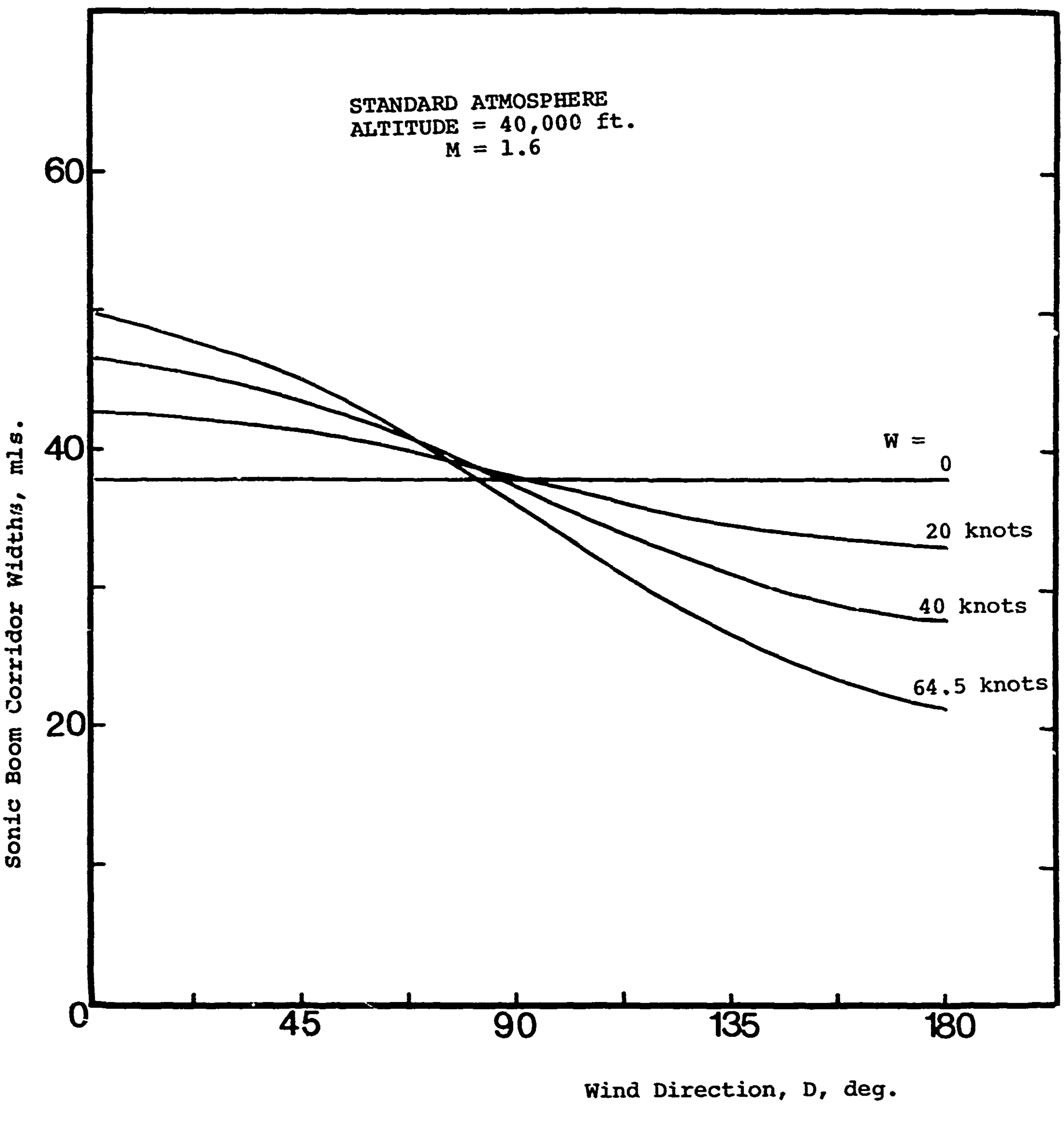


FIG. 5. EFFECTS OF WIND MAGNITUDE AND DIRECTION ON SONIC BOOM CORRIDOR.

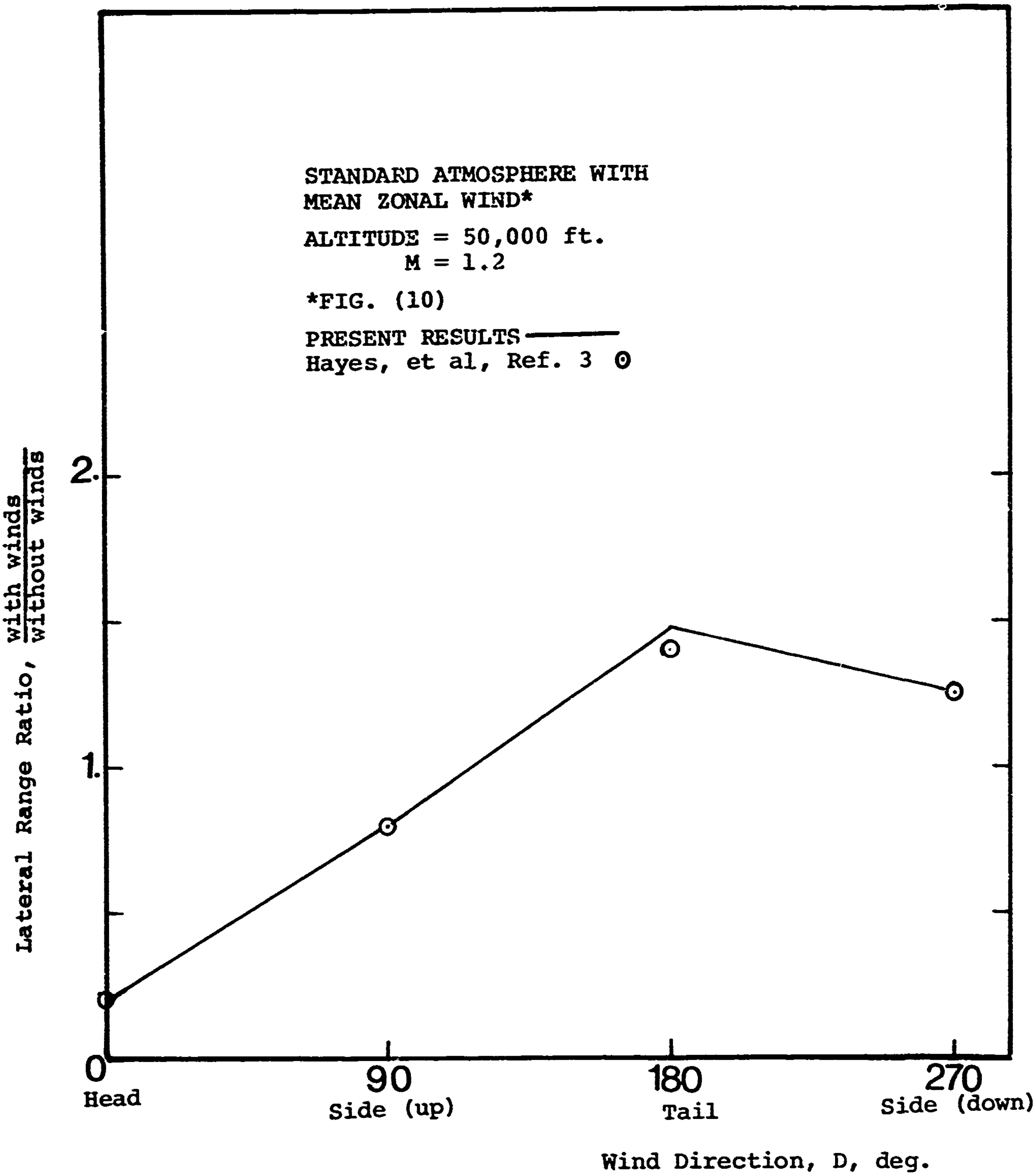


FIG. 6. COMPARISON OF PRESENT RESULTS WITH PUBLISHED DATA.

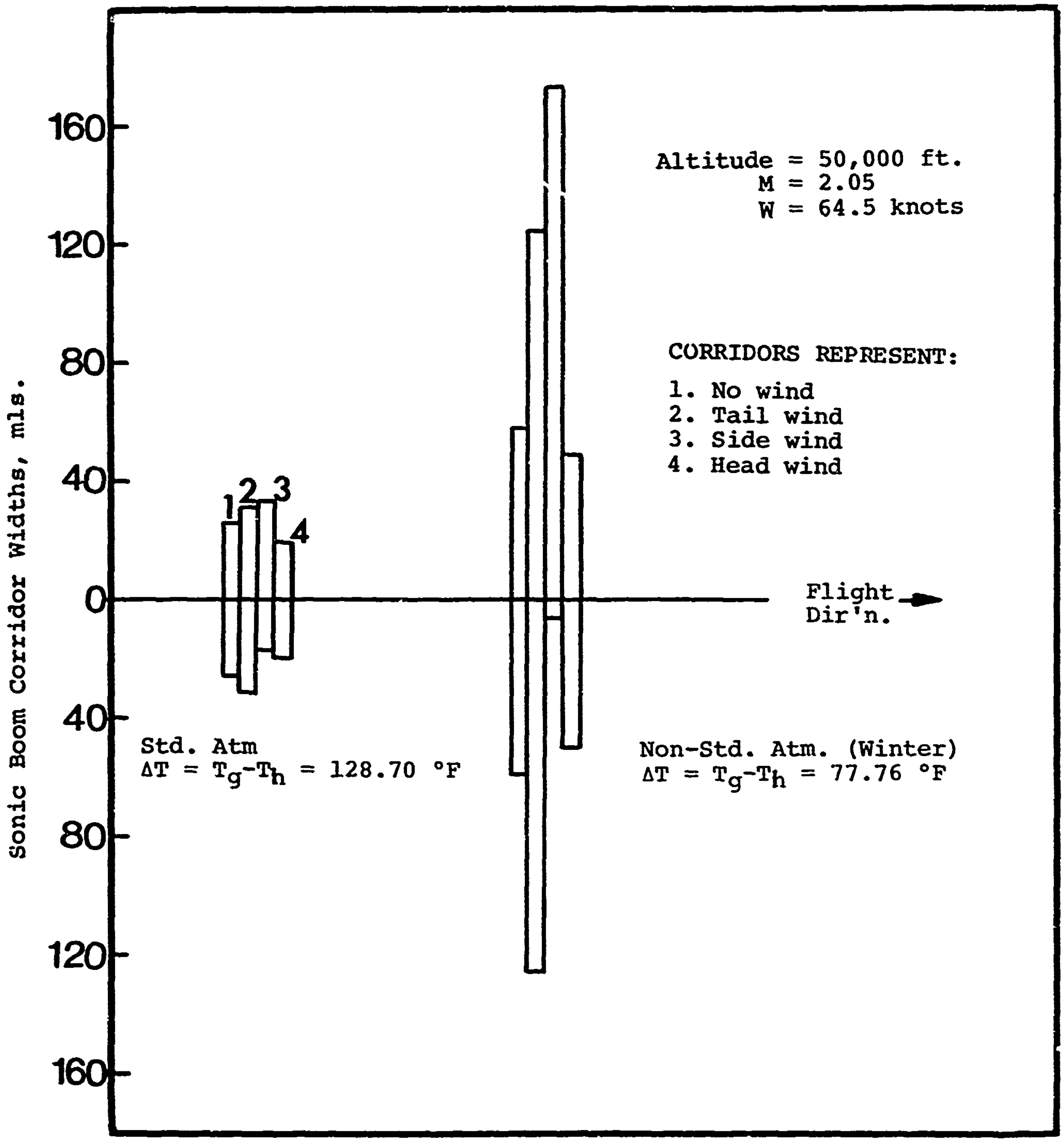


FIG. 7. EFFECTS OF GROUND TEMPERATURE ON SONIC BOOM CORRIDOR.

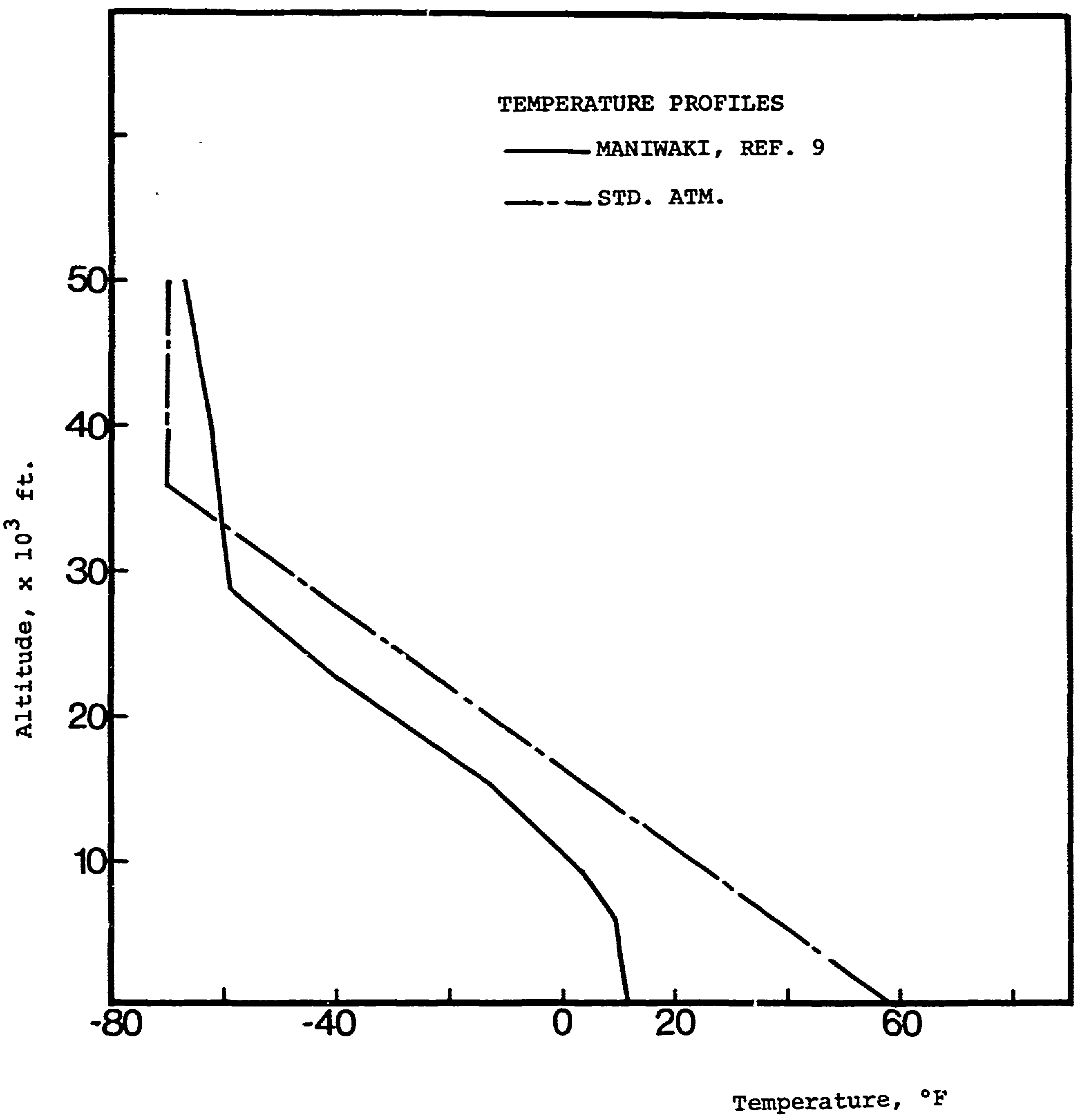


FIG. 8. STANDARD AND WINTER ATMOSPHERIC TEMPERATURE PROFILES.

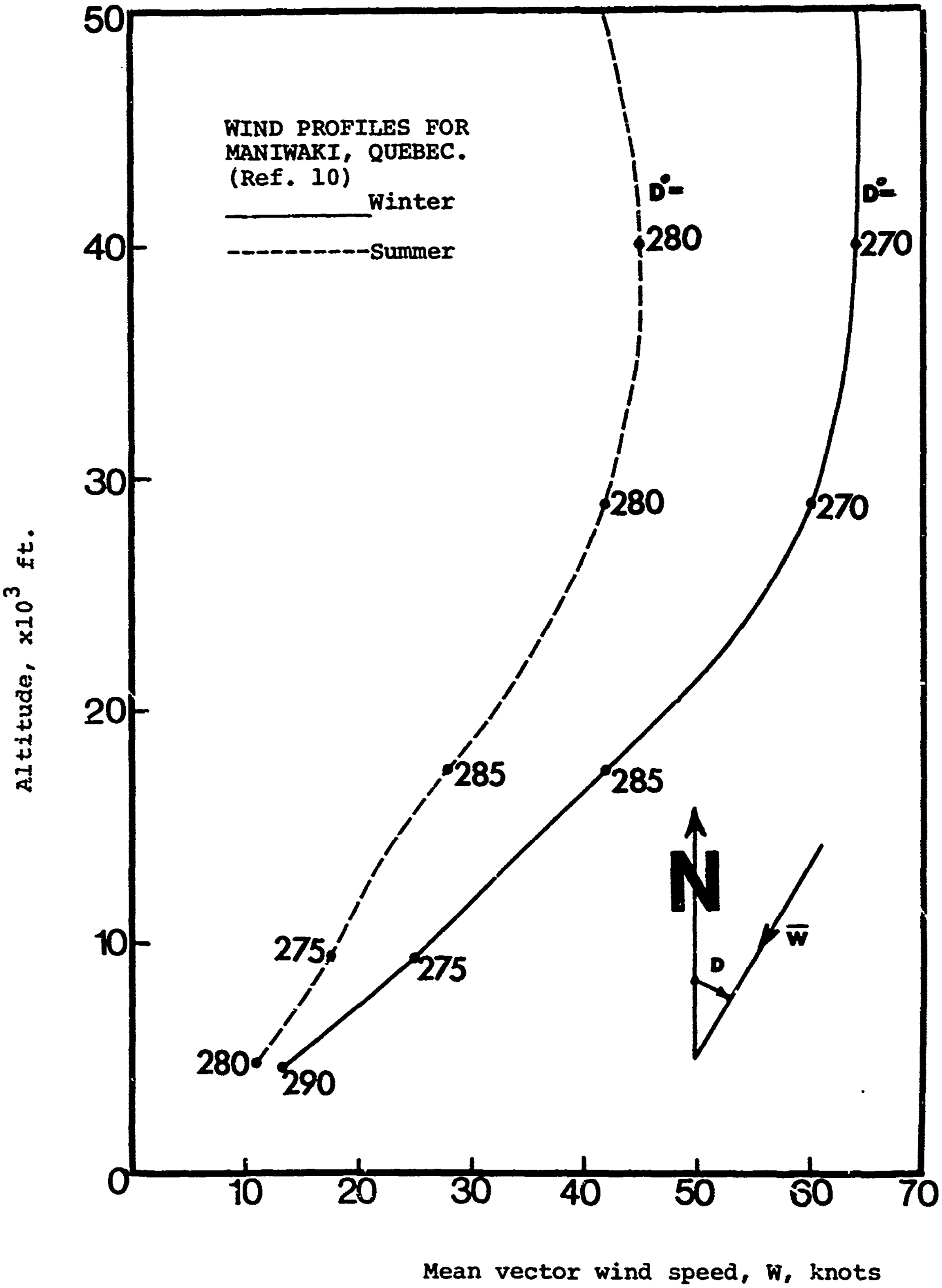


FIG. 9. SUMMER AND WINTER MEAN VECTOR WINDS FOR MANIWAKI, QUEBEC.

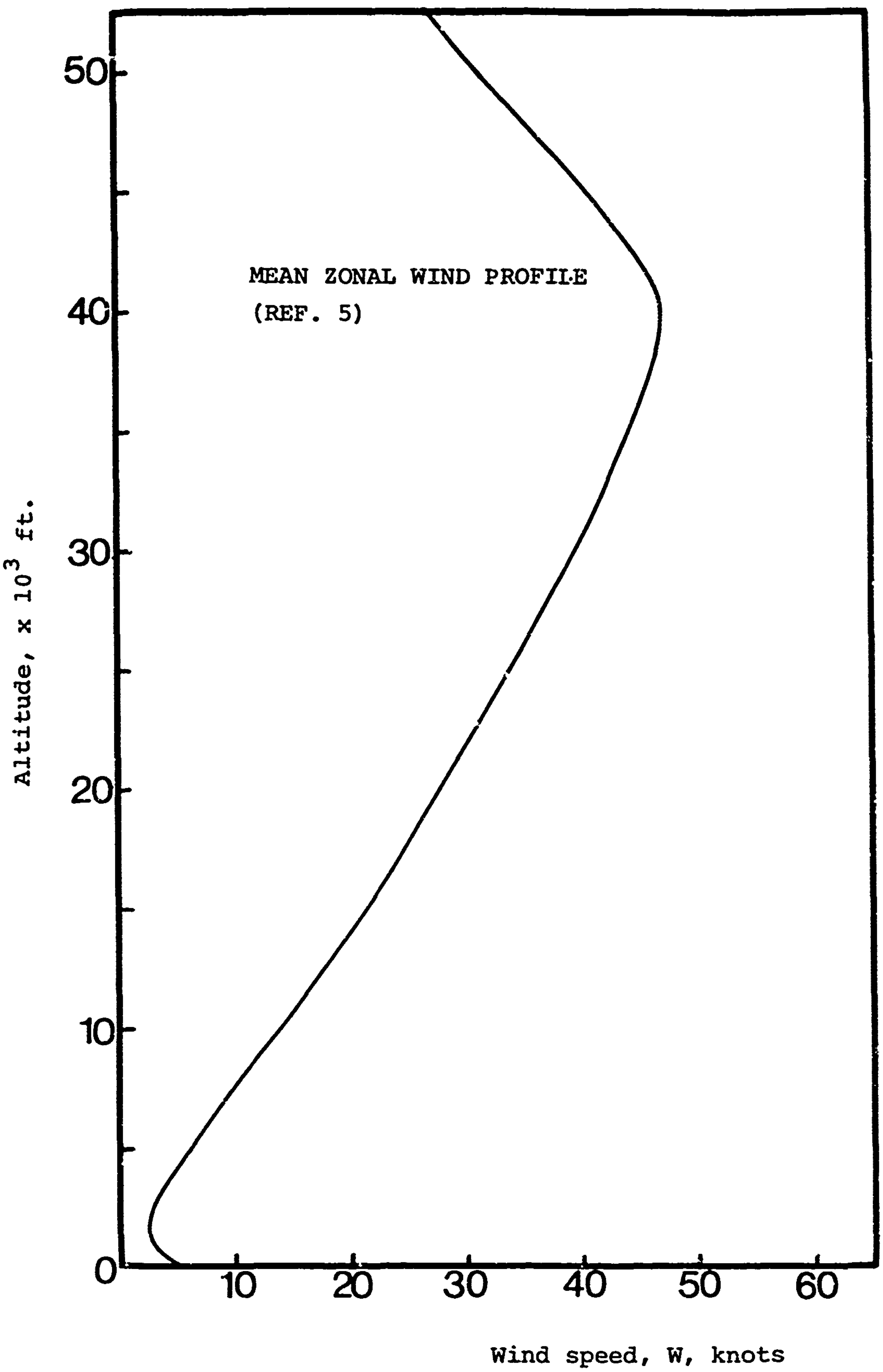


FIG. 10. MEAN ZONAL WIND PROFILE.